DISCRETE TIME SIGNALS AND SYSTEMS

2 Discrete-Time Signals and Systems

- 2.1 Discrete-Time Signals
- 2.2 Discrete-Time Systems
- 2.3 Analysis of Discrete-Time LTI Systems
- 2.4 Discrete-Time Systems Described by Difference Equations
- 2.5 Implementation of Discrete-Time Systems
- 2.6 Correlation of Discrete-Time Signals

2.2.1 Input-Output Description of Systems

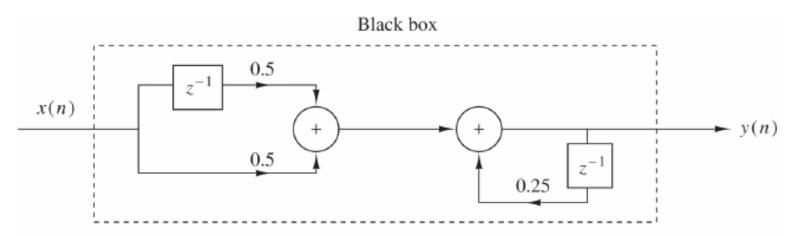
Ex. Determine the response of the following systems to

$$x(n) = \{ \dots 0, 0, -3, -2, -1, 0, 1, 2, 3, 0, 0, \dots \}$$

- $1. \quad y(n) = x(n)$
- 2. y(n) = x(n-1)
- 3. y(n) = x(n+1)
- 4. y(n) = [x(n+1) + x(n) + x(n-1)]/3
- 5. y(n) = median [x(n + 1), x(n), x(n 1)]
- 6. $y(n) = x(n) + x(n-1) + x(n-2) + \dots$ = y(n-1) + x(n)

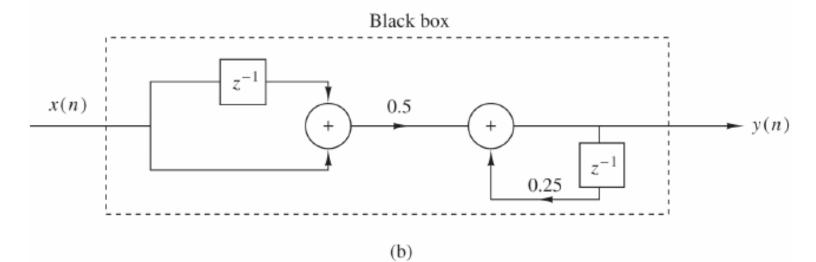
2.2.2 Block Diagram Representation of Discrete-Time Systems





$$y(n) = 0.25 y(n-1) + 0.5 x(n) + x(n-1)$$

Initial condition?



2.2.3 Classification of Discrete-Time Systems

- 1. Static (memoryless) versus dynamic: y(n) = T[x(n), n]
- 2. <u>Time-invariant versus time-variant:</u> $x(n-k) \rightarrow y(n-k)$
- 3. <u>Linear versus nonlinear</u>: $T[a_1x_1(n) + a_2x_2(n)] = a_1T[x_1(n)] + a_2T[x_2(n)]$
- 4. <u>Causal versus noncausal:</u> y(n) = F[x(n), x(n-1), x(n-2), ...]
- 5. <u>Stable versus unstable:</u> $|x(n)| \le M_x < \infty \rightarrow |y(n)| \le M_y < \infty$

2.2.3 Classification of Discrete-Time Systems

Ex. Time-invariant versus time-variant: $x(n-k) \rightarrow y(n, k) \stackrel{?}{=} y(n-k)$

1.
$$y(n) = x(n) + x(n-1)$$

$$2. \quad y(n) = nx(n)$$

3.
$$y(n) = x(-n)$$

4.
$$y(n) = x(n) \cos \omega_0 n$$

$$y(n, k) = x(n-k) + x(n-k-1) = y(n-k)$$

$$y(n, k) = n x(n-k) \neq y(n-k)$$

$$y(n, k) = x(-n-k) \neq y(n-k)$$

$$y(n, k) \neq y(n-k)$$

2.2.3 Classification of Discrete-Time Systems

Ex. Linear versus nonlinear: $T[a_1x_1(n) + a_2x_2(n)] \stackrel{?}{=} a_1T[x_1(n)] + a_2T[x_2(n)]$

1.
$$y(n) = nx(n)$$

2.
$$y(n) = x(n^2)$$

3.
$$y(n) = x^2(n)$$

4.
$$y(n) = Ax(n) + B$$

5. $y(n) = e^{x(n)}$

$$5. \quad y(n) = e^{x(n)}$$

Let
$$x(n) = a_1 x_1(n) + a_2 x_2(n)$$
 and

Check
$$T[x(n)] = a_1 T[x_1(n)] + a_2 T[x_2(n)]$$

2.2.3 Classification of Discrete-Time Systems

Ex. Causal versus noncausal: $y(n) \stackrel{?}{=} F[x(n), x(n-1), x(n-2), ...]$

1.
$$y(n) = x(n) + x(n-1)$$

$$2. \quad y(n) = \sum_{k=-\infty}^{n} x(k)$$

3.
$$y(n) = ax(n)$$

4.
$$y(n) = x(n) + 3x(n + 4)$$

$$5. \quad y(n) = x(n^2)$$

6.
$$y(n) = 2x(n)$$

7.
$$y(n) = x(-n)$$

2.2.3 Classification of Discrete-Time Systems

Ex. Stable versus unstable: $|x(n)| \le M_x < \infty$? $|y(n)| \le M_y < \infty$

$$y(n) = y^2(n-1) + x(n)$$

$$x(n) = \{ ...0, 0, 2, 0, 0, ... \}$$

$$\downarrow y(n) = \{ ...0, 0, 2, 2^2, 2^4, 2^8, ... \}$$

2.3 Analysis of Discrete-Time LTI Systems

2.3.2 Resolution of a Discrete-Time Signal into Impulses (1/2)

